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Supersymmetric Wilson Loops, Superstring-like Observables, and the Natural Coupling of Superstrings To Supersymmetric Gauge Theories.

M. Awada[+] and F.Mansouri[*]
Physics Department
University of Cincinnati,Cincinnati, OH-45221

Abstract

We obtain an explicit expression for the supersymmetric Wilson loop in terms of chiral superfields and supercurrents in superspace. The result turns out to be different from what one would expects from the simple replacement of Lie algebraic valued connection in the exponent with the corresponding super-Lie algebraic one. In the abelian supersymmetric gauge theories, generalizing the super-particle coupling represented by the exponent of the supersymmetric Wilson loop, we show there exist a unique dimensionless coupling of the superstring to abelian supersymmetric gauge theories that respects all known symmetries. The coupling is expressed in terms of chiral currents in superspace. The natural superstring coupling gives rise to a new observable that is "stringy" in nature and has no analogue in non-supersymmetric gauge theories.

+ E-Mail address: moustafa@physunc.phy.uc.edu
∗ E-Mail address: Mansouri@UCBEH.SAN.UC.EDU

Wilson loop observables [1,2] have found a variety of applications in the study of gauge theories, ranging from phenomenology to topological field theories [3]. This is because they are gauge invariant observables characterized by a dimensionless coupling constant. In abelian gauge theories they can be interpreted quantum mechanically as the amplitude of the creation and annihilation of e^-e^+ resulting from the unique coupling between the Maxwell's field and the point particle with dimensionless coupling constant. In non-abelian gauge theories, this interpretation can be extended formally, in the sense that the particle-Yang-Mills coupling can only be realized by the Wilson loop

$$W_R(C) = TrPe^{ie\oint_C A} \tag{1}$$

in a given representation R of the Lie algebra. In equation (1) the symbol Tr stands for the trace, P for path ordering, and C is a closed path. At short distance scales perturbative calculations shows that the Wilson loop produces the perimeter law indicating short range finite forces between quarks as well as the renormalization of the Coulomb potential between two heavy static quarks. These results indicate that quarks can propagate freely at short distances. On the other hand in lattice gauge theories strong coupling expansion shows that the Wilson loop produces the Area law suggesting confinement. Unfortunately there is no known analytical method of extracting the linear potential or the area law from the Wilson loop in the continuum. This raises the question whether the area law or the string can arise naturally through some dimensionless coupling to gauge theories. It can be easily seen that there is no consistent dimensionless coupling with at most two derivatives between strings and gauge theories except for the parity violating expression

$$ie\int d\sigma F^* \; . \tag{2}$$

In (2) $d\sigma$ is the surface measure, and F^* is the dual of the gauge field strength. However, there do exist higher derivative dimensionful couplings between strings and gauge fields [4].

In the last two years, there have been significant developments in N=1 and N=2 supersymmetric gauge theories [5]. In particular a mechanism for confinement was provided by the condensation of monopoles. In finite supersymmetric gauge theories, such as N=4 super Yang-Mills theory, and a class of N=2 super Yang-Mills theories coupled to N=2 matter the corresponding beta functions vanish and the issues of confinement and asymptotic freedom become unclear. Therefore, it is of interest to explore whether the supersymmetric extension of the Wilson loop or the existence of new supersymmetric observables might shed some light on the above issues.

The aim of this letter is two fold: One is to generalize the usual notion of a Wilson loop to the supersymmetric case. The other is to

show that, unlike the non-supersymmetric gauge theories, there is a new observable in supersymmetric gauge theories whose current is chiral and has support only on a two surface with a boundary. Its generalization to an arbitrary surface gives rise to a unique and natural coupling between the superstring and the supersymmetric gauge theories, which is characterized by a dimensionless coupling constant. This allows us to construct a new "stringy" observable, in addition to the supersymmetric Wilson loop, that respects all the symmetries of the theory.

We begin with the supersymmetric Wilson loop. For clarity of presentation, we give the details for the supersymmetric Maxwell's theory. The generalization to the supersymmetric non-abelian gauge theories will be presented in a second article. Throughout this paper we will use the superspace two component notation.

In the expression (1) for the Wilson loop, the notion of a connection plays a dominant role. So it might appear at first sight that to obtain a suitable extension of (1) for the supersymmetric gauge theories, we only need to replace the Lie algebra valued connection with a superalgbera valued connection and replace the trace instruction with the supertrace one. Although such a prescription works in some special cases for locally supersymmetric gauge theories, such as Chern-Simons supergravity theories in 2+1 dimensions [6], it does not work for globally supersymmetric gauge theories. The underlying reason is the structure of the superpartners of the gauge potential in the two cases. The superpartner of the former carries a vector index, while that of the latter does not.

To look for another point of departure, we return to the exponent of the Wilson loop which has the structure of the interaction between the point particle and the abelian gauge field represented uniquely by a dimensionless coupling constant e:

$$S_{int} = ie \oint_C d\lambda A(x(\lambda)).\dot{x}(\lambda) \ . \tag{3}$$

Stoke's theorem implies that the interaction lagrangian can be rewritten in terms of the field strength of the gauge field on a closed two surface Σ whose boundary is the loop C:

$$S_{int} = \frac{ie}{2} \int_{\Sigma(C)} d^2\xi \epsilon^{ab} F_{ab} \tag{4}$$

where $\xi^a; a = 1, 2$ is the coordinate of the two world sheet, and [4]

$$F_{ab} := v^\mu_a v^\nu_b F_{\nu\mu} = \partial_a A_b - \partial_b A_a \tag{5}$$

where the field A_a is defined to be the projection of the gauge field $A_\mu; \mu = 0..3$

along the two surface:

$$A_a := v_a^\mu A_\mu \ \ ; \ \ v_a^\mu = \partial_a x^\mu(\xi) \ . \tag{6}$$

It is important to remark that the Wilson loop is a topological entity. Furthermore the fact that there is no difference between the two definitions (3) and (4) of the Wilson loop can be interpreted to mean that the extra surface coordinate, say, ξ^1, is not dynamical and can be integrated over.

The Supersymmetric Wilson loop

The important advantage of expressing the SUSY Wilson loop in terms of superfields is that it will be manifestly supersymmetric and gauge invariant. We start by generalizing the interaction (4) to that of a superparticle coupled to a supersymmetric abelian gauge field theory. The definition in (6) can be generalized in superspace using a super one-form Γ_A and the standard supersymmetric notation:

$$F_{ab} = v_a^A v_b^B F_{BA} \tag{7}$$

where

$$F_{AB} = D_{[A}\Gamma_{B)} - T_{AB}^C \Gamma_C \tag{8}$$

and T is the torsion supertensor. The bases v have to be constructed from the super-coordinate $z^M = (x^\mu, \theta^m, \theta^{\dot m})$ in such a way that they are supersymmetric invariant. We define

$$v_a^A = E_M^A \partial_a z^M \tag{9a}$$

where E_M^A is a veilbein and obtain the following components:

$$v_a^{\alpha\dot\alpha} = \partial_a x^{\alpha\dot\alpha}(\xi) - \frac{i}{2}(\theta^\alpha(\xi)\partial_a\theta^{\dot\alpha}(\xi) + \theta^{\dot\alpha}(\xi)\partial_a\theta^\alpha(\xi))$$

$$v_a^\alpha = \partial_a \theta^\alpha(\xi) \tag{9b}$$

$$v_a^{\dot\alpha} = \partial_a \theta^{\dot\alpha}(\xi).$$

The v's are invariant under the global space-time supersymmetry transformation

rules defined

$$\delta x^{\alpha\dot{\alpha}}(\xi) = \frac{i}{2}(\epsilon^{\alpha}\theta^{\dot{\alpha}}(\xi) + \epsilon^{\dot{\alpha}}\theta^{\alpha}(\xi))$$

$$\delta\theta^{\alpha}(\xi) = \epsilon^{\alpha} \tag{9c}$$

$$\delta\theta^{\dot{\alpha}}(\xi) = \epsilon^{\dot{\alpha}}.$$

The requirement that the coordinates θ satisfy the Majorana condition demands that ϵ be a Majorana. From the v's we will construct the following supersymmetric and gauge invariant tensors:

$$C_{ab}^{\dot{\alpha}\dot{\beta}} = \frac{i}{2}v_{a\beta}^{(\dot{\alpha}}v_{b}^{\dot{\beta})\beta} \quad ; C_{ab}^{\alpha\beta} = \frac{i}{2}v_{a\dot{\beta}}^{(\alpha}v_{b}^{\beta)\dot{\beta}} \tag{10a}$$

$$C_{ab}^{\alpha} := v_{a}^{\alpha\dot{\alpha}}v_{b\dot{\alpha}} \tag{10b}$$

$$C_{ab}^{\dot{\alpha}} := v_{a}^{\alpha\dot{\alpha}}v_{b\alpha}.$$

The vector components of the C's are antisymmetric in a and b by construction (the parenthesis denotes symmetrization with unit weight factor) and obey the following properties:

$$C_{ab}^{\dot{\alpha}\dot{\beta}} = \frac{1}{2}\epsilon_{ab}C^{\dot{\alpha}\dot{\beta}} \quad ; \quad C^{\dot{\alpha}\dot{\beta}} = \epsilon^{ab}C_{ab}^{\dot{\alpha}\dot{\beta}}$$

$$C_{ab}^{\alpha\beta} = \frac{1}{2}\epsilon_{ab}C^{\alpha\beta} \quad ; \quad C^{\alpha\beta} = \epsilon^{ab}C_{ab}^{\alpha\beta}\ . \tag{10c}$$

Note, however, the spinor components of C do not obey any particular symmetry. This crucial property allows us to use these components to construct a new type of interaction with supersymmetric gauge theories that are

inherently "stringy" as we will show below. For later use we also note that

$$C_{[ab]}^{\alpha} = \frac{1}{2}\epsilon_{ab}C^{\alpha} \quad ; \quad C^{\alpha} = \epsilon^{ab}C_{ab}^{\alpha}$$

$$C_{[ab]}^{\dot{\alpha}} = \frac{1}{2}\epsilon_{ab}C^{\dot{\alpha}} \quad ; \quad C^{\dot{\alpha}} = \epsilon^{ab}C_{ab}^{\dot{\alpha}} \tag{10d}$$

where the square bracket indicates antisymmetrization. To get the correct content of fields of super Maxwell's theory or super Yang-Mills theory one has to impose

representation preserving constraints whose purpose is to project the superfield formulation onto chiral superspace where the supersymmetry representation is irreduciable. The required constraints are [7] (and references therein):

$$F_{\alpha\dot{\alpha}} = F_{\alpha\beta} = F_{\dot{\alpha}\dot{\beta}} = 0 \ . \tag{11a}$$

Together with the Bianchi identities of F_{AB}, one can determine the remaining components of F in terms of a chiral superfield $(W_\alpha, W_{\dot{\alpha}})$. For the abelian supergauge theory one obtains:

$$F_{\alpha,\beta\dot{\beta}} = \epsilon_{\alpha\beta} W_{\dot{\beta}} \ \ ; \ \ F_{\dot{\alpha},\beta\dot{\beta}} = \epsilon_{\dot{\alpha}\dot{\beta}} W_\beta$$

$$F_{\alpha\dot{\alpha},\beta\dot{\beta}} = \frac{-i}{2}(\epsilon_{\alpha\beta} D_{(\dot{\alpha}} W_{\dot{\beta})} + \epsilon_{\dot{\alpha}\dot{\beta}} D_{(\alpha} W_{\beta)}) \tag{11b}$$

where the W's satisfy the chirality conditions:

$$D_\alpha W_{\dot{\beta}} = D_{\dot{\alpha}} W_\beta = 0 \tag{12a}$$

and

$$D^{\dot{\alpha}} W_{\dot{\alpha}} = D^\alpha W_\alpha \ . \tag{12b}$$

In turn the W's are determined in terms of an unconstrained vector superfield V:

$$W_\alpha = \frac{-i}{2} \bar{D}^2 D_\alpha V \ \ ; \ \ W_{\dot{\alpha}} = \frac{i}{2} D^2 D_{\dot{\alpha}} V \ . \tag{13}$$

They are invariant under the gauge transformation

$$\delta V = i(\bar{\Lambda} - \Lambda) \tag{14}$$

where Λ $(\bar{\Lambda})$ is a chiral (anti-chiral) parameter superfield. The component expansion of V and W_α in the Wess-Zumino gauge are respectively,

$$V = (0, 0, 0, 0, A_{\alpha\dot{\alpha}}, \psi_\alpha, \psi_{\dot{\alpha}}, D) \tag{15a}$$

$$W_\alpha = \psi_\alpha - \theta^\beta f_{\alpha\beta} - i\theta_\alpha D + \frac{i}{2}\theta^2 \partial_{\alpha\dot{\alpha}} \psi^{\dot{\alpha}} \tag{15b}$$

where ψ is the superpartner of the gauge field $A_{\alpha\dot{\alpha}}$, $f_{\alpha\beta} = \frac{1}{2}\partial_{(\alpha\dot{\alpha}} A^{\dot{\alpha}}_{\beta)}$ is the Maxwell's field strength and D is an auxiliary field. Another important property of the W_α

$(W_{\dot\alpha})$ is that it is invariant under the chiral (anti-chiral) supersymmetry transformations of the component fields:

$$\delta_{\epsilon^\alpha} W_\alpha = \delta_{\epsilon^{\dot\alpha}} W_{\dot\alpha} = 0 \ . \tag{16a}$$

$$\delta A^{\alpha\dot\alpha} = i(\epsilon^\alpha \psi^{\dot\alpha} + \epsilon^{\dot\alpha}\psi^\alpha)$$

$$\delta\psi_\alpha = \epsilon^\beta f_{\alpha\beta} + i\epsilon_\alpha D \tag{16b}$$

$$\delta D = \frac{1}{2}\partial_{\alpha\dot\alpha}(\epsilon^\alpha\psi^{\dot\alpha} - \epsilon^{\dot\alpha}\psi^\alpha)$$

Equipped with (11) and using the tensor definitions (10) we deduce from (8) that the interaction action between the superparticle and the supersymmetric gauge theory is given by

$$S^{(1)}_{int} = \frac{1}{2} ie \int\limits_{\Sigma(C)} d^2\xi \epsilon^{ab} F_{ab} \ . \tag{17a}$$

But now the field strength F_{ab} is given by

$$F_{ab} = \epsilon_{ab}(\frac{1}{2}C^{\alpha\beta}(\xi)D_\alpha W_\beta(x(\xi),\theta(\xi)) + C^\alpha(\xi)W_\alpha(x(\xi),\theta(\xi)) + h.c) \tag{17b}$$

where h.c denotes hermitian conjugate. We will shortly show that (17) can be neatly rewritten in terms of chiral currents in superspace. The abelian supesymmetric Wilson loop is now given by

$$W(C) = e^{S^{(1)}_{int}} \tag{18}$$

A Superstring-like Observable

Looking at the structure of the spinor components of C in (10) one immediately recognizes the existence of a new two form which is not topological but a dynamical interaction that has support only on the surface:

$$S^{(2)}_{int.} = \kappa \int\limits_{\Sigma(C)} d^2\xi \sqrt{-h} h^{ab} C^{\alpha}_{ab}(\xi) W_{\alpha}(x(\xi), \theta(\xi)) + h.c \tag{19}$$

where h^{ab} is the metric on the two surface and h is its determinant. We know of no way to construct such a term in the absence of supersymmetry. Moreover, unlike the expression (18), this interaction cannot be reduced to an expression on the line. It is supersymmetric, gauge, and reparemetrization invariant and characterized by a new dimensionless coupling constant κ which is different from e classically. Thus we can define a new super-gauge invariant observable

$$\Psi(C) = e^{S^{(2)}_{int.}} \tag{20}$$

whose correlation function might be of potential interest for strongly coupled super QED.

Chiral Currents

The combination of the Wilson loop and the superstring-like observable can be totally expressed in terms of chiral currents on the surface. Define

$$J^{\alpha}(z) = q^{\alpha}(z) - D_{\alpha} q^{\alpha\beta}(z) \tag{21a}$$

where

$$q^{\alpha}(z) = \int\limits_{\Sigma(C)} d^2\xi (eC^{\alpha}(\xi) + \kappa\sqrt{-h} h^{ab} C^{\alpha}_{ab}(\xi))\delta^6(z - z(\xi)) \tag{21b}$$

$$q^{\alpha\beta}(z) = \frac{1}{2} \int\limits_{\Sigma(C)} d^2\xi (eC^{\alpha\beta}(\xi))\delta^6(z - z(\xi)) \tag{21c}$$

and $\delta^6(z - z(\xi)) = \delta^4(x - x(\xi))(\theta^2 - \theta^2(\xi))$ is the chiral superspace delta function. The full interacting actions (17) and (19) are therefore given by:

$$S_{int} = S^{(1)}_{int} + S^{(2)}_{int} = \int d^6 z (J^{\alpha} W_{\alpha} + h.c) \ . \tag{22}$$

which is manifestly supersymmetric and gauge invariant. We could have discovered the interacting lagrangians (17) and (19) by postulating (22) on basis of dimen-

sionality, gauge, and supersymmetric invariance. The dimension of W is $[W] = -\frac{3}{2}$ in units of length. Since $[d^6 z] = +3$ we deduce that $[J] = -\frac{3}{2}$. If J is a chiral current with support on the two surface

$$J^\alpha(z) = \int\limits_{\Sigma(C)} d^2\xi j^\alpha(\xi)\delta^6(z - z(\xi)) \tag{23}$$

then $j^\alpha(\xi)$ is a two dimensional supersymmetric and reparametrization invariant spinor of dimension $-\frac{1}{2}$. The only candidate objects to construct such a $j^\alpha(\xi)$ are from the tensors C in equation (10). In particular the spinor components have the correct dimension of $-\frac{1}{2}$. The most general structure of $j^\alpha(\xi)$ is

$$j^\alpha(\xi) = \kappa\sqrt{-h}h^{ab}C^\alpha_{ab}(\xi) + \tilde{e}C^\alpha(\xi) + \frac{e}{2}C^{\alpha\beta}(\xi)D_\alpha \tag{24}$$

where the super-derivative is understood to act on the chiral delta function in (23). The fact that the surface has a boundary, together with the Bianchi identity (12b) and current conservation implies that $e = \tilde{e}$, however if the surface is closed with no boundary, then e and $\tilde{e}$ are different in general. Of course in the absence of supersymmetry $C^\alpha_{ab} = 0$ and the current (24) reduces to that of the Wilson loop. Having obtained the supersymmetric Wilson loop and a superstring-like observable, we note that (19) can be viewed as a new type of interaction that corresponds to the interaction of a superstring with the abelian supersymmetric gauge theory if the chiral current (23) is defined over an arbitrary surface without a boundary. In this context, equation (22) represents the interaction of both the superparticle and the superstring with an abelian supersymmetric gauge theory characterized by dimensionless coupling constants. Integrating out the gauge field from the expression (19) will lead to an effective superstring theory as a consequence of short distance renormalization. A natural question which arises is whether such a superstring theory is one of the known varieties or is a new one. We will have more to say about this elsewhere [8]. Another important question is whether such a superstring theory or a superstring-like observable can exist in a $D+1$ dimensional Minkowski world of the form M_DxR. This existence depends crucially on whether or not the closed surface Σ can be embedded in M_D. For $D = 3$ the answer is certainly negative in general.

However, for simply connected 3-mainfolds such as the three sphere,

the embedding exists. Furthermore as shown and discussed in [9], there is a whole class of non-simply connected manifolds in which any loop C embedded in them can be thought of as the boundary of a closed surface Σ in M_3.

To summarize, the main results of this work may be stated, a posteriori, in the following way. We have obtained a general gauge and supersymmetric invariant expressions for the interaction of the superparticle and the superstring with a supersymmetric abelian gauge field. When restricted to a surface with a boundary, these expressions lead to the supersymmetric Wilson loop and a superstring-like observable which has no counter part in non-supersymmetric gauge theories.

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References

[1] K.G.Wilson, Phys.Rev. D10 (1974) 2455.

[2] A.M. Polyakov, Phys. Lett. B59 (1975) 82; F. Wegner, J.Math. Phy. 12 (1971) 2259

[3] E. Witten, Comm. Math. Phys. 117, 353 (1988).

[4] L.N.Chang, and F. Mansouridvi2ps - x 10 susy —lpr -Plw4 proceeding of the John Hopkins workshop, ed. G.Domokos and S.Kovesi Domokos, John Hopkins Univ. (1974).

[5] N. Sieberg and E. Witten, Nucl. Phys. B426 19 (1994).

[6] K. Koehler, F. Mansouri, C. Vaz, and L. Witten, Nucl. Phys. B348 ,373, (1990).

[7] J. Wess and J. Bagger, Introduction to supersymmetry, Princeton University Press 1983.

[8] M. Awada and F. Mansouri, in preperation.

[9] M.Awada, Comm. Math. Phys. 129;329 (1990).